TDC-I
P-I
Gaseous State

Boyle Temperature

The Temperature at which the real gas obeys Boyle's Law over a wide range of pressure is called Boyle Temperature (T_B)

- T_B is characteristics of each real gas which depends upon Vander Waal gas constant a & b

For 1mole of real gas, Equation of State may be written as

$$\left(P + \frac{a}{V^2}\right)(V-b) = RT$$

$$P = \frac{RT}{V-b} - \frac{a}{V^2} \quad \text{--- (1)}$$

Multiplying bothside by V, we have

$$PV = \frac{RTV}{V-b} - \frac{a}{V} \quad \text{--- (II)}$$

Differentiating eqn (II) w.r.to P at constant Temperature

$$\left(\frac{\delta(PV)}{\delta P}\right)_T = RT\left[\frac{1}{V-b}\left(\frac{\delta V}{\delta P}\right)_T + \frac{V^{-1}}{(V-b)^2}\left(\frac{\delta V}{\delta P}\right)_T\right] + \frac{a}{V^2}\left[\frac{\delta V}{\delta P}\right]_T$$

$$= RT\left[\frac{1}{V-b} - \frac{V}{(V-b)^2}\right]\left[\frac{\delta V}{\delta P}\right]_T + \frac{a}{V^2}\left[\frac{\delta V}{\delta P}\right]_T$$

$$\left[\frac{\partial(PV)}{\partial P}\right]_T = \left[\frac{RT}{V-b} - \frac{RTV}{(V-b)^2} + \frac{a}{V^2}\right]\left(\frac{\partial V}{\partial P}\right)_T$$

Since $\left(\frac{\partial V}{\partial P}\right)_T$ is always negative

hence it is not Equal to zero.

if $\left[\frac{\partial(PV)}{\partial P}\right]_T = 0$

$$\frac{RT}{V-b} - \frac{RTV}{(V-b)^2} + \frac{a}{V^2} = 0$$

$$RT\left[\frac{1}{V-b} - \frac{V}{(V-b)^2}\right] = \frac{-a}{V^2}$$

$$RT\left[\frac{V-b-V}{(V-b)^2}\right] = \frac{-a^2}{V^2}$$

$$RT\left[\frac{-b}{(V-b)^2}\right] = \frac{-a}{V^2}$$

$$RT = \frac{a}{b}\left(\frac{V-b}{V}\right)^2$$

$$RT = \frac{a}{b}\left(1 - \frac{b}{V}\right)^2$$

At Boyle Temp, $T = T_B$

$$T_B = \frac{a}{Rb}\left[1 - \frac{b}{V}\right]^2$$

Since $P \to 0$, $\frac{b}{V} \to 0$ Hence $\boxed{T_B = \frac{a}{Rb}}$